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The focusing properties of inhomogeneous magnetic sector fields

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With 1 figure in the text

1. Introduction

Homogeneous magnetic sector fields find many applications for analyzers of electron or ion beams. In this paper some advantages of using inhomogeneous sector fields are pointed out. In the first place the most important property of both kinds of fields is that of separating ions of different momenta. In the second place one is interested to achieve geometrical focusing in order to obtain an intense and sharp image. As for the first of these properties both types of fields have about the same qualities; but with respect to geometrical focusing it seems preferable to use inhomogeneous fields in many cases where homogeneous fields are used at present.

It is especially noteworthy that cylinder symmetrical fields with so called two-directional focusing can be constructed with nearly the same ease as homogeneous fields with the essential limitation of a fixed radius of curvature of the beam.

2. The laws of image formation

A ray, proceeding from the central line $r = r_0$ in a radial direction in a magnetic field whose axial component in the symmetry plane of the field is $H(r)$, crosses the central line after an angle ϕ_r, the radial focusing angle, determined by

$$\Phi_r = \pi \left[1 + \frac{r_0 H'(r_0)}{H(r_0)} \right]^{-\frac{1}{2}} \tag{1}$$

while for another ray, proceeding in an axial direction, the (axial) focusing angle is

$$\Phi_z = \pi \left[- \frac{r_0 H'(r_0)}{H(r_0)} \right]^{-\frac{1}{2}}. \tag{2}$$

Since any orbit in the vicinity of the central line is governed by two linear noncoupled differential equations, namely for the radial deviation δ_r from the circle $r = r_0$.

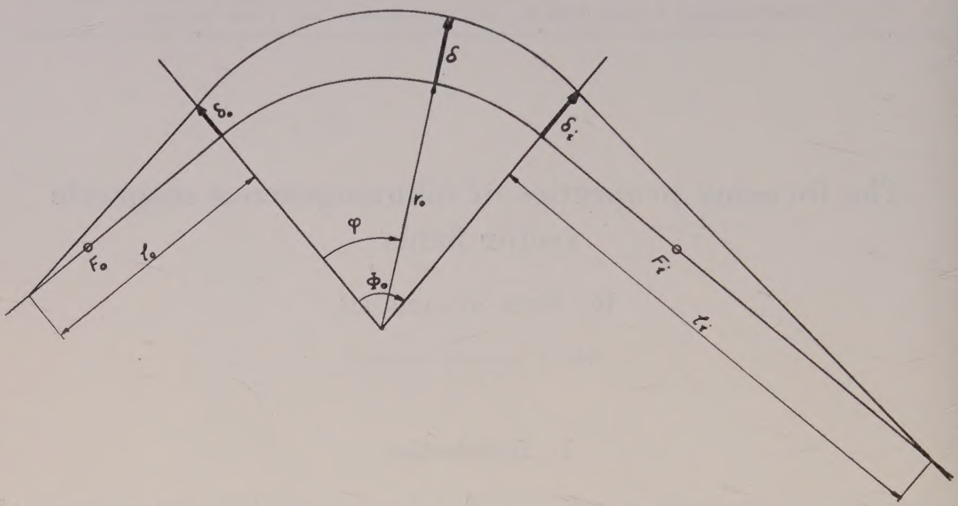

Fig. 1.

$$\frac{d^2\,\delta_r}{r_0^2\,d\,\varphi^2} + \frac{\pi^2}{\Phi_r^2}\,\delta_r = 0 \tag{3 a}$$

and, similarly, for the axial deviation δ_z,

$$\frac{d^2\,\delta_z}{r_0^2\,d\,\varphi^2} + \frac{\pi^2}{\Phi_z^2}\,\delta_z = 0 \tag{3 b}$$

where φ is the angular distance from an arbitrary point, we may apply the ordinary laws of first-order image formation for each of two rays, one deviating radially, the other axially from the central line.

Let us assume that (see Fig. 1) the magnetic field has the shape of a sector with the centrum angle ϕ_0 and that the field is sharply cut off by axial planes from the field-free space. Our task is then to find the correlations between those straight lines of the object and image space, which pertain to the same orbit. Since the calculations for δ_r and δ_z are completely analogous, we may drop the index.

A definite ray traversing the entrance plane of the magnet, determined e. g. by $\varphi = 0$, is given by the values of δ and $d\,\delta/\delta\,\varphi$, namely δ_0 and δ_0' for $\varphi = 0$. In an arbitrary axial plane within the magnet, especially the exit plane $\varphi = \Phi_0$, the values of the same entities are computed from the differential equations (3) obtaining

$$\delta = \delta_i = \delta_0 \cos \pi\,\phi_0/\phi + (\delta_0'\,\phi/\pi) \sin \pi\,\phi_0/\phi \tag{4}$$

$$d\,\delta/d\,\varphi = \delta_i' = -\,(\delta_0\,\pi/\phi) \sin \pi\,\varphi_0/\phi + \delta_0' \cos \pi\,\phi_0/\phi. \tag{5}$$

If the distance of the object from the entrance plane $\varphi = 0$ is l_0 and if, similarly, the distance of the image from the exit plane is l_i we get the relations

$$\delta_0'/r_0 = \delta_0/l_0 \tag{6}$$

$$\delta_i'/r_0 = -\delta_i/l_i. \tag{7}$$

Eliminating δ_0, δ_0', δ_i, δ_i' between Eqs. (4)—(7) we obtain

$$l_0 l_i = (l_0 + l_i) r_0 \cot \pi \phi_0/\phi + r_0^2 \phi/\pi. \tag{8}$$

This equation shows that the focus, on the image as well as on the object side, is situated at the distance

$$d = r_0 \cot \pi \phi_0/\phi \tag{9}$$

from its nearest boundary plane and counted positively outwards from the same plane. Introducing the object and image distances from the respective foci,

$$s_0 = l_0 - d, \qquad s_i = l_i - d \tag{10}$$

we may write Eq. (8) as follows

$$s_0 s_i = r_0^2 (\phi/\pi + \cot^2 \pi \phi_0/\phi). \tag{11}$$

Consequently the focal distance f is

$$f = r_0 (\phi/\pi + \cot^2 \pi \phi_0/\phi)^{\frac{1}{2}}. \tag{12}$$

This equation together with Eq. (9) determines the positions of the principal planes of the system.

In the case of exact two-directional focusing, when according to Eqs. (1)—(2)

$$\phi_r = \phi_z = \pi \sqrt{2} \tag{13}$$

the formulae (9) and (12) simplify to

$$d = r_0 \cot \phi_0/\sqrt{2} \tag{14}$$

$$f = r_0 [\sqrt{2} + \cot^2 \phi_0/\sqrt{2}]^{\frac{1}{2}}. \tag{15}$$

After a simple transformation, the formulae (9) and (12) may be written in the same form as for homogeneous fields. Inserting

$$r_0 = r_0' \sqrt{\pi/\phi} \tag{16}$$

$$\phi_0 = (\phi/\pi) \operatorname{arccot} (\sqrt{\phi/\pi} \cot \phi_0') \tag{17}$$

in Eqs. (10) and (13) we obtain

$$d = r_0' \cot \phi_0' \tag{18}$$

$$f = r_0'/\sin \phi_0'. \tag{19}$$

Consequently the image construction can be performed in the same way as for homogeneous sector fields if the parameters r_0 and ϕ_0 are transformed into

r_0' and ϕ_0' by means of Eqs. (16) and (17). In the case of stigmatic image formation the latter equations simplify into

$$r_0 = r_0' \sqrt[4]{2} \tag{20}$$

$$\phi_0 = \sqrt{2}\ \mathrm{arccot}\ (\sqrt[4]{2} \cot \phi_0'). \tag{21}$$

3. Applications

In an apparatus where the needs of very high resolution and a large solid angle are not claimed at one and the same time it is permissible to neglect the focusing action of the fringing fields at the entrance and the exit of the beam into the sector field. Under the same circumstances one is allowed to shape the field in a number of ways compatible with the condition for two-directional focusing. Among these fields a particular one will be stressed here, namely

$$H(r) = H_0 \cdot 2\, r_0 / (r + r_0) \tag{22}$$

since it can be realized by using *conical* pole-faces which is very convenient from a constructional point of view. Assuming the permeability of the iron to be infinite the pole distance $2z$ as a function of the radius r is expressed by

$$2z = z_0 (r + r_0) / r_0 \tag{23}$$

where z_0 is the value of z at the central line $r = r_0$. All other fields of two-directional focusing can be realized by means of pole faces having the last mentioned cones as tangent cones at $r = r_0$.

The present ideas were developed in 1947 in response to a request for a deflector magnet intended for a van de Graaff-machine at Försvarets Forskningsanstalt, Stockholm.

REFERENCES: **N. Svartholm** and **K. Siegbahn,** Ark. f. matem. astr. o. fysik, *33 A*, No 21, 1946 and Nature. — **N. Svartholm** Ark. f. matem. astr. o. fysik, *33 A*, No 24, 1946.

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